*Applied Mechanics and Materials Vols. 789-790 (2015) pp 735-741*

*doi:10.4028/www.scientific.net/AMM.789-790.735*

# Modeling and Control of a Robot Arm on a Two Wheeled Moving Platform

Mert Onkol[1,a], Cosku Kasnakoglu[1,b]

[1]Department of Electrical and Electronics Engineering, TOBB University of Economics and Technology, Turkey, Tel: +903122924064

[a]mert.onkol@etu.edu.tr, [b]kasnakoglu@etu.edu.tr

**Keywords:** robot arm on a two wheeled moving platform, series PID, cascaded PID, LQG

**Abstract.** In this paper, the derivation of dynamic model of a robot arm on a two wheeled moving platform, and design of controllers to stabilize the robot arm are presented. The modeling of two wheeled moving platform is conducted through Simmechanics® toolbox of Matlab® software. Considered control approaches are PID control and linear quadratic gaussian (LGQ) for the dynamic system. The controllers are designed by using linearized model devised from Simmechanics®. Simulation studies are discussed. Control approaches are compared in detail in terms of tracking precision, quality of control signal. The aims of this study are derivation of linearized model for designing controllers, and determining the most appropriate controller for the real time system.

## Introduction

Mobile robots have been major area of research interest in both educational and industrial purposes [1,2]. The substantial advantages of wheeled mobile systems over legged robots are ease of design, agility, increased mobility, efficiency of energy, and does not need complex control structures and joints to manipulate the robotic structure [3,4]. Mobile systems are utilized in different types of environments and applications including industrial, military, security, household . Mobile platforms with two wheels can be considered as dynamically balancing structure, and generally called as inverted pendulum robot. Comparison of two wheel platform against three wheel one could be feasible because when the number of wheels are more than three, the mobile platform requires suspension to move on uneven surfaces which brings more complex mechanical design and cost. Although three wheeled system is statically stable, two wheeled system has high maneuverability and small fingerprint which leads to utilize robot in narrow and small environments [5].

Robot arm or manipulators are another widely used robotic application in daily life and industry. The main application areas are assembly, handling at machine tools, welding, pick and place work etc.. [6,7]. Robot arm is a sort of mechanical system, has similar functionality to a human arm. Robot arm can perform rotational or translational motion regarding the structure. The links of arm are connected to each other by joints, and actuation is applied through joints by generally dc servo motors to conduct tasks. Robot arm and two wheeled mobile platform are two separate frames considering the mechanical structure and tasks that achieved. Unification of these two structures emerges a brand new concept which can be classified as robot arm on a two wheeled moving platform. Small footprint and maneuverability features of two wheeled platform and sophisticated task achievement capabilities of robot arm makes the entire system inevitably advantageous. The whole system can be considered as inverted pendulum as aforementioned above, and should be dynamically balanced due to highly nonlinear structure of the system [8].

Modeling and control subjecting robot arm on two wheeled platforms have been investigated so far. Most of the modeling studies depends on Euler-Lagrange derivation and Newton- Euler approaches [9,10]. Studies generally focus on designing controllers to balance the mobile system. Nasir et.al propose LQR controller and evaluation of performance is in terms of input tracking and disturbance rejection [11]. Almeshal et. al. designs inverted pendulum robot with additional degrees of freedom to extend working space [12]. Lee and Jung achieve balancing and navigation of inverted pendulum robot with low cost sensors. System can navigate through predefined waypoints while keeping the inverted pendulum balanced [13]. Pathak et. al. derive firstly dynamic equation regarding wheel torques as input of the system. Partial linearizartion is applied to nonlinear system to design controllers. Velocity controller tracks the desired path while stabilizing the inverted pendulum's angle [14].

In this paper PID and LQG control schemes are utilized to stabilize the angle and translational position of the mobile platform. PID control signal is constructed regarding current error value, integral term of error and derivative of error. Although the PID controller gives adequate performance measures on mobile system, linear quadratic gaussian (LQG) controller is an optimal control scheme which is a combination of Kalman filter and linear quadratic regulator (LQR). LQG is designed based on devised linear time invariant state space equation. Apart from conventional controller LQG controller is a dynamical system as the controlled system. Modeling of the mobile platform is achieved on Simmechanics® to obtain most appropriate design solution for real time system.

This paper is proceeds as follows: The second part derives the dynamic equation of the mobile robot system via Simmechanics® environment. The third part PID and LQG control schemes are designed respectively by utilizing Matlab® software. The fourth part considers comparison of control schemes. Finally, conclusion on simulation studies and future work is given.

## System Modeling

Derivation of dynamic model is the major part of the system control. Because performance of the two wheeled platform is based on the quality of control approach and dynamic model. Two wheels are directly mounted on the chassis of the platform. By moving back and forth via applying torque on each wheel, the vehicle balances itself to upright position. To move in any direction $x$, the vehicle should change inclination angle $\theta$ as seen in Fig. 1. In order to derive most reliable dynamic model, vehicle is modeled on Simmechanics® environment. In Fig. 1, visual output of the system can be seen clearly.

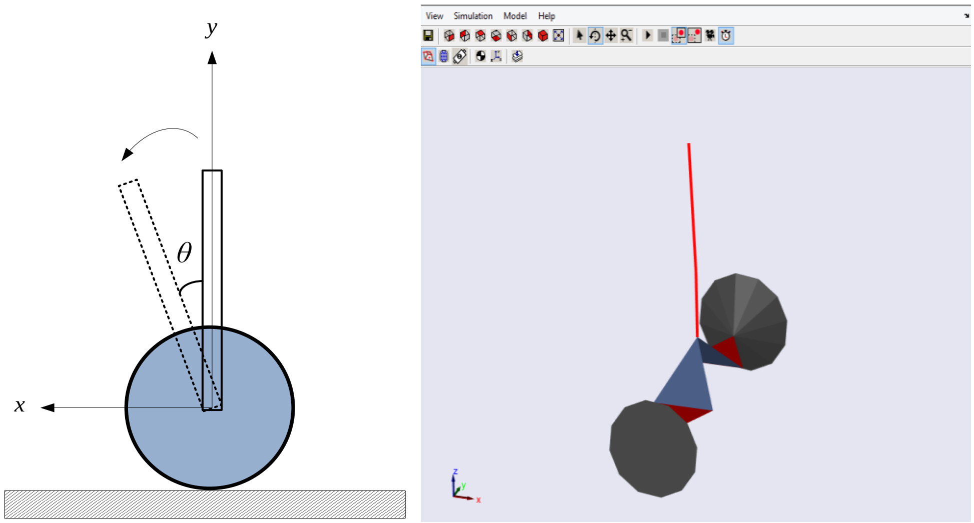


Fig. 1. Model (a) and Simmechanics (b) construction of robot arm on two wheeled moving platform

Although the dynamic model is nonlinear, it is linearized around equilibrium points ($x = 0$, $\theta = 0$). Defining the states of systems as $X = [x,\ \dot{x},\ \theta,\ \dot{\theta}]$, the state space equations can be written as follows:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & A_{12} & 0 & 0\\ 0 & 0 & A_{33} & A_{24}\\ 0 & 0 & 0 & A_{34}\\ 0 & 0 & A_{43} & A_{44}\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ B_{21}\\ 0\\ B_{41}\end{bmatrix}u \quad , \quad y = \begin{bmatrix}C_{11} & 0 & 0 & 0\\ 0 & 0 & C_{23} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}u \qquad (1)$$

where $A_{12}$ =1, $A_{33}$ = - 0.0137, $A_{24}$ = 0.020, $A_{34}$ = 1, $A_{43}$ = 1.8192, $A_{44}$ = - 2.6563, $B_{21}$ = 0.0378, $B_{41}$ = - 0.070, $C_{11}$ = 1, $C_{23}$ = 57.2958 which are parameters of linearized vehicle dynamics.

**Control Approaches**

The major purpose of the control design of the system is to track specified trajectory by keeping the robot arm balanced or tracking a reference angle generated to track a path. Due to the coupled characteristics of the system dynamics, designing a control approach could be tough objective. The number of degrees of freedom is more than control inputs, these types of systems are called as underactuated mechanical systems. Coupled characteristics of dynamics affects the system performance directly because tracking task is intensively depends on the precision of the balancing control performance of the robot arm. In this study, three linear control schemes are utilized to control the system by means of tracking and balancing. These are PID and LQG control approaches.

**PID Control Scheme.** Although PID control is a linear control method, it is widely used in control of nonlinear dynamic systems effectively. Two types of PID control structures are mentioned in this study. These are parallel PID control scheme and cascaded PID control scheme.
As seen from Fig. 2, the decoupled PID controllers are utilized. First controller is for controlling the angle around equilibrium point, second controller for controlling the linear position around equilibrium point. Cascaded PID controller is also depicted in Fig.2. Outer position controller generates appropriate reference angle values for PID angle controller.

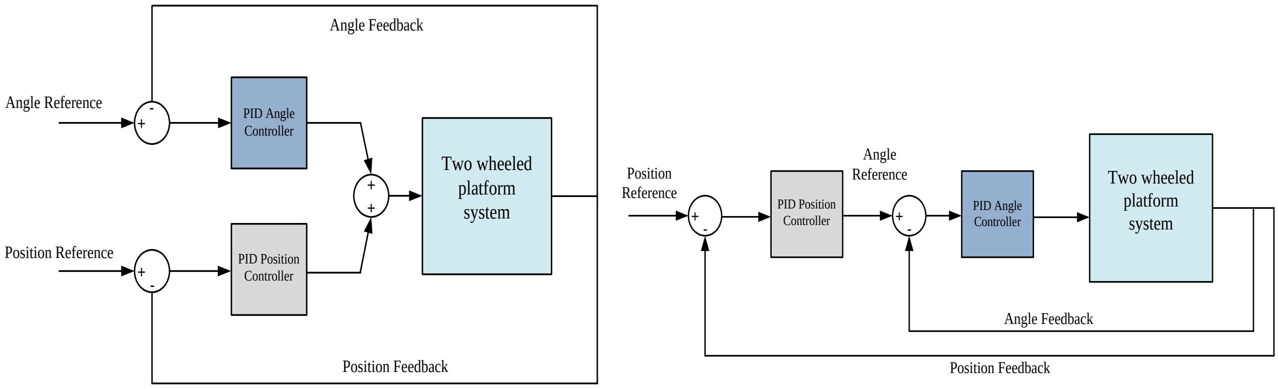

Fig. 2. Parallel PID structure and Casceded PID structure

**Parallel PID Control Scheme.** Tuning process was carried out by utilizing PID control tuning function of Matlab® Simulink®. Firstly, PID controller for the fast dynamics which is angle, then PID controller for position, was designed respectively. After the design phase, following parameter values for the controllers are evaluated as in Table 1.

Table 1: Parallel PID parameters

| | $K_P$ | $K_I$ | $K_D$ | Filter Coeff (N) |
|---|---|---|---|---|
| Angle Controller | -77.306 | -8.721 | -28.862 | 28.637 |
| Position Controller | -7.853 | -0.055 | -32.488 | 0.451 |

After the simulation was run on Matlab® , the following results are obtained in Fig. 3. The simulation is run with following parameters:

Table 2: Simulation parameters

| | |
|---|---|
| Initial angular position | 10° |
| Initial linear position | 0 meters |
| Simulation time | 500 seconds |
| Reference angular position | 0° |
| Reference linear position | 0 meters |

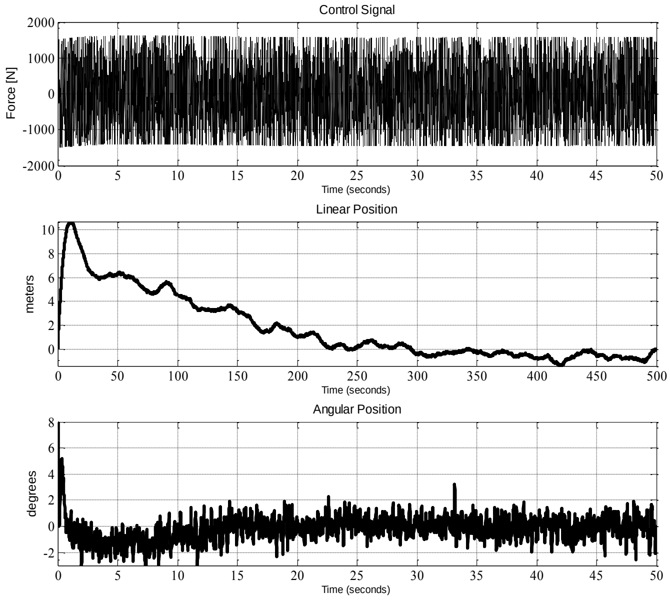


Fig. 3. Control signal and tracking results for parallel PID control scheme

The results of the controller are shown in Fig. 3. Initial angular error is fairly compensated about in 10 seconds. However, linear position reached approximately 10 meters away from the initial value. It takes considerable much time to recovery to its desired value. The control signal fluctuates inside its predefined range.

**Cascaded PID Control Scheme.** Due to the poor performance indices of series PID controller, more power PID scheme should have been considered. Cascade PID controller involves two control loops that use two feedback signals to control one essential variable. In our system, primary controller is position controller that generates reference angle values for the secondary controller. The control signal of secondary controller is directly applied to system. Although, the secondary controller controls fast dynamics, the primary controller handles slow dynamics of the system. In Table. 3 controller parameters are depicted.

Table 3: Parallel PID parameters

| | $K_P$ | $K_I$ | $K_D$ | Filter Coeff (N) |
|---|---|---|---|---|
| Angle Controller | -77.306 | -8.721 | -28.862 | 28.637 |
| Position Controller | 0.426 | 0.002 | 1.319 | 2.067 |

The simulation studies were carried out under same conditions as in series PID except simulation time of 50 sec.

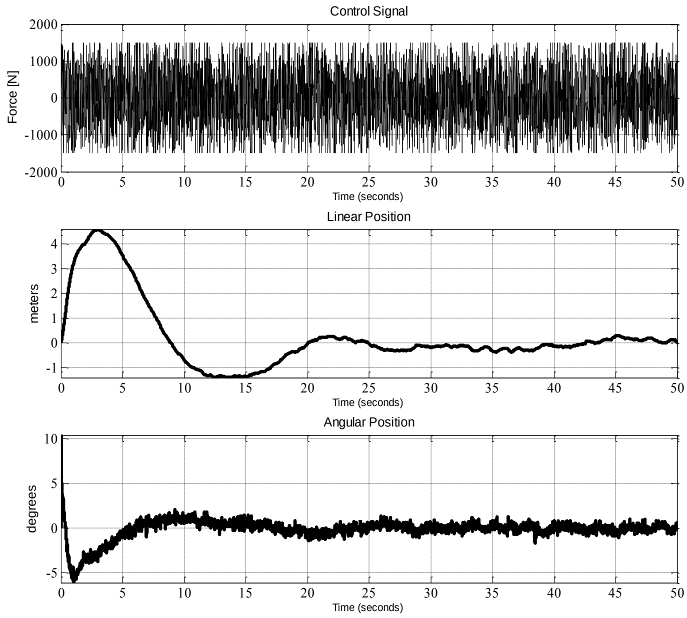


Fig. 4. Control signal and tracking results for cascaded PID control scheme

As it can be clearly seen from Fig. 4, unlike the results in series PID simulations, linear position and angular position reaches their steady state values in reasonable time. Position controller generates required reference angle values to stabilize itself. Linear position values do not reach to such values in previous simulation study. Angular position values are changing inside reasonable magnitudes which are reference values generated by position controller. The controller produces nearly smooth control signals with respect to previous one.

**LQG Control Scheme.** Linear Quadratic Gaussian control is combination of Linear Quadratic Regulator (LQR) and Kalman filter. LQG guarantees the stability of the closed-loop system. Performance of the system can be focused by confidence of system stability. In order design LQG controller for two wheeled system, states of the system are *x*, $\dot{x}, \theta, \dot{\theta}$. LQG regulator minimizes the cost function in Eq. 2.

$$J = \left\{ \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^{\tau} [x^T, u^T] Q_{xu} \begin{bmatrix} x \\ u \end{bmatrix} dt \right\} \tag{2}$$

depending on the state equations in Eq. 3 where the *w* is process noise and *v* is measurement noise. *R* and *Q* values in Eq. 4 were tuned properly to minimize cost function in Eq. 2.

$$\begin{aligned} \dot{x} &= Ax + Bu + w \\ y &= Cx + Du + v \end{aligned} \tag{3}$$

$$R = 0.001\,,\ Q = 0.1 \tag{4}$$

Designed LQG dynamic system is as follows:

$$\begin{bmatrix} \dot{\hat{x}} \\ \ddot{\hat{x}} \\ \dot{\hat{\theta}} \\ \ddot{\hat{\theta}} \end{bmatrix} = \begin{bmatrix} -1.7321 & 1 & -0.0155 & 0 \\ 0.1950 & 6.1881 & 161.0684 & 50.1104 \\ 0 & 0 & -57.499 & 1 \\ -0.2216 & -1.1459 & -39.7365 & -11.9323 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \dot{\hat{x}} \\ \hat{\theta} \\ \dot{\hat{\theta}} \end{bmatrix} + \begin{bmatrix} 1.7321 & 0.0003 \\ 1 & 0.001 \\ 0 & 1.0036 \\ 0.0003 & 0.2045 \end{bmatrix} [x \quad \theta]^T \tag{5}$$

$$y = [31.6 \quad 163.8 \quad 4264.3 \quad 1325] \begin{bmatrix} \hat{x} \\ \dot{\hat{x}} \\ \hat{\theta} \\ \dot{\hat{\theta}} \end{bmatrix} + [0 \quad 0][x \quad \theta]^T$$

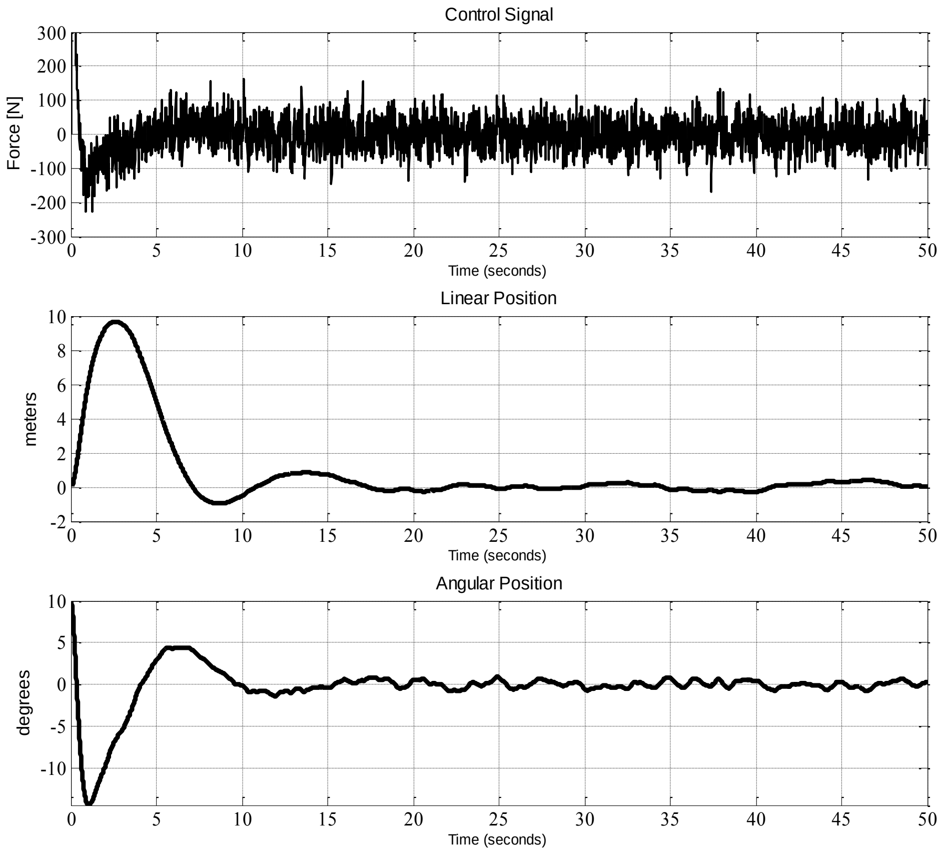


Fig. 5. Control signal and tracking results for LQG control scheme

The results of LQG control scheme in Fig. 5 that linear position and angular position are regulated with satisfactory precision. Although the linear position exceeds the error value in cascaded PID, angular position quickly regulates itself. On the other hand, the control signal is quiet smoother than the PID equivalent, nearly one tenth of previous signals.

## Conclusion

Three different approaches have been considered on dynamic model derived in Matlab® software. Series PID control scheme, cascaded PID control scheme and LQG control designs are applied respectively. The advantage of both PID controllers is that they do not need a detailed dynamic model of the system. Controllers could be adjusted via trial and error process. However, by the help of powerful tuning capabilities of Matlab® environment, most appropriate controller gains are set to yield the most performance from system. Especially, cascaded PID controller has better performance by means of control signal and regulation time even the original plant is nonlinear and coupled. LQG control design is an optimal control approach which is also suitable for linear dynamic systems. Although the LQG linear position and angular position dynamics have not the best overshoot performance, the control signal magnitude and smoothness is the most compatible one by means of applying to real time system. In essence, both three controllers are capable of controlling the nonlinear unstable robot arm on two wheeled moving platform. In future studies, these control techniques are going to be implemented on real time model.

## References


[1] J. H. Lee and H. Hashimoto, "Controlling Mobile Robots in Distributed Intelligent Sensor Network," *IEEE Trans. on Industrial Electronics*, vol. 50, pp. 890-902, October 2003.

[2] M. Pinto, A. P. Moreira, and A. Matos, "Localization of Mobile Robots Using an Extended Kalman Filter in a LEGO NXT," *IEEE Trans. on Education*, vol. 55, pp. 135-144, Feb. 2012.

[3] E. Takeuchi and T. Tsubouchi, "Sensory Data Processing Middlewares for Service Mobile Robot Applications," in *SICE-ICASE International Joint Conf.*, 2006, pp. 1201-1206.

[4] H. Utz, S. Sablatnög, S. Enderle, and G. Kraetzschmar, "Miro-Middleware for Mobile Robot Applications," IEEE *Trans. on Robotics and Automation*, vol. 18, pp. 493-497, August 2002.

[5] R.P. M. Chan, K. A. Stol, and C.R. Halkyard, "Review of Modelling and Control of two-wheeled Robots," *Anual Reviews in Control*, vol. 37, pp. 89-103, 2013.

[6] K. Sun and V. Lumelsky, "Path Planning Among Unknown Obstacles: The Case of a Three-Dimensional Cartesian Arm," *IEEE Trans. on Robotics and Automation*, vol. 8, pp. 776-786, Dec. 1992.

[7] J. Kofman, X. Wu, T. J. Luu, and S. Verma, "Teleoperation of a Robot Manipulator Using a Vision-Based Human-Robot Interface," *IEEE Trans. on Industrial Electronics*, vol. 52, pp. 1206-1219, Oct. 2005.

[8] F. O. Silva and L. H. C. Ferreira, "Design and Implementation of a PID Control System for a Coaxial Two-Wheeled Mobile Robot," in *IEEE International Symposium on Industrial Electronics* (ISIE), pp.1- 6, 2013.

[9] A. N. K. Nasir, M. A. Ahmad, R. Ghazali, and N. S. Pakheri, "Performance Comparison Between Fuzzy Logic Controller (FLC) and PID Controller for a Highly Nonlinear Two-Wheels Balancing Robot," in *First International Conference on Informatics and Computational Intelligence*, 2011 ,pp. 176-181.

[10] J, X, Xu, Z. Q. Guo, and T. H. Lee, "Design and Implementation of Integral-Sliding Mode Control on an Underactuated Two-Wheeled Mobile Robot," *IEEE Trans. on Industrial Electronics*, vol. 61, pp. 3671-3681, July 2014.

[11] A.N.K. Nasir, M.A. Ahmad, and R. M. T. Raja Ismail, "The Control of Highly Nonlinear Two Wheels Balancing Robot: A Comparative Assessment Between LQR and PID-PID Control Schemes," in *World Academy of Science, Engineering and Technology*, vol. 4, pp. 176-181, 2010.

[12] A.M. Almeshal, K. M. Goher, and M.O. Tokhi, "Dynamic Modelling and Stabilization of a New Configuration of Two-Wheeled Machines," *Robotics and Automation Systems*, vol. 61, pp. 443-472, 2013.

[13] H. Lee and S. Jung, "Balancing and Navigation Control of a Mobile Inverted Pendulum Robot Using Sensor Fusion of Low Cost Sensors," *Mechatronics*, vol. 22, pp. 95-105, 2012.

[14] K. Patnak, J. Franch, and S. Agrawal, "Velocity and Position Control of a wheeled Inverted Pendulum by a partial Feedback Linearization," *IEEE Trans. on Robotics*, vol. 2, pp. 505-513, June 2005.