A Nonlinear Dynamical Modeling and Control Method for the Vorticity Control of the Flow past a Circular Cylinder

H. Deniz Karaca, G. Deniz Özen, and Coşku Kasnakoğlu

***Abstract*—In this paper a systematic approach for the nonlinear modeling and feedback control of vorticity behind an immersed circular cylinder system was developed. In this technique first a number of control input points over the cylinder and some measurement points for vorticity past the cylinder are assigned. A type of nonlinear dynamic model (namely a Hammerstein-Wiener (HW) model) of the flow field is estimated via system identification techniques using measurement data obtained from a chirp input function. Once the dynamical model of the system is estimated, a controller for the linear block of the HW model is designed using internal model control method, and this controller is then mapped to the HW model by reversing the input/output nonlinearity functions. The procedure described is implemented and tested numerically in MATLAB and CFD computations performed on the closed-loop system show that the controller is capable of achieving significant reduction in the vorticity levels past the cylinder.**

***Index Terms*—Flow past a circular cylinder, nonlinear dynamical modeling and control, system identification, hammerstein-wiener method, vorticity control.**

I. Introduction

One of the popular active research topics in fluid flow systems is analyzing and controlling the flow around a circular cylinder which contains vortex shedding, turbulent behavior at low Reynolds numbers and an unsteady wake region. For many years the flow around a circular cylinder has been considered as a benchmark problem for the understanding, modeling and control of more complex fluid problems because of its simple geometry and typical behavior of separated flows [1], [2]. Understanding and analyzing the behavior of the flow around a circular cylinder mathematically is possible by representing it with dynamical models. Navier-Stokes (NS) partial differential equations (PDEs) are the most accurate way to represent this flow but they are complex, difficult to analyze and solve analytically [3]. To simplify these PDEs model reduction techniques such as Proper Orthogonal Decomposition and Galerkin Projection have been used to obtain simpler representations of the flow. Studies on such model reduction approaches include Noack *et al*. [4], [5] who proposed a model reduction strategy for Galerkin models and by adding shift modes they achieved accurate representations of the unstable solution for the cylinder wake. Tadmor *et. al.* [6] carried out studies to include dynamic mean field representations in low order Galerkin models. Another strategy to obtain simplified models of the flow process is to use empirical deductions directly from computational fluid dynamics (CFD); e.g. Apaçoğlu *et. al.* [7], [8] investigated uncontrolled and controlled turbulent and laminar flow over a circular cylinder using such a direct empirical approach.

In literature one also finds many techniques with the purpose of shaping the past a cylinder. For instance, Fujitsawa *et. al.* [9] controlled vortex shedding behind a rotating cylinder by designing a phase lead-lag compensator. Modi [10] performed experiments intended to achieve moving surface boundary layer control of airfoils. Homescu *et. al.* [11] studied an optimal control approach for the active control of incompressible viscous flow past a circular cylinder. Fagley *et. al*. [12] investigated nonlinear adaptive regulation of the vortex shedding phenomenon for aero-optic applications. Aamo *et. al.* [13] designed a feedback controller for the global asymptotic stabilization of a Ginzburg-Landau model of vortex shedding Aleksic *et. al*. [14] proposed a nonlinear control strategy using a low-dimensional Galerkin model which was applied by the help of a transverse local volume force. Many additional control approaches such as the prevention of transition by using objects in suitable form, surface cooling or heating and injection and suction of fluids are also available in literature [15].

This paper we propose a systematic approach to produce a nonlinear model and controller for a circular cylinder system. The approach differs from the abovementioned literature in the sense that the nonlinear models produced are obtained directly by an input/output system identification approach, without requiring complex manipulations of the governing NS PDEs. In addition, the controller design on the nonlinear model is performed by exploiting the special structure of a HW model, where an LQG controller designed for the inner linear part is later mapped to the entire HW model through reversing input/ouput nonlinearities. The rest of the paper deals with the details of methodology, as well as numerical simulation results.

II. Methodology

A. CFD Simulations for Gathering Input-Output Data

For a certain range of Reynolds numbers the flow past cylinder forms vortices which are periodic and swirling in opposite directions. The first step is to obtain simulations for this phenomenon. For this purpose we use Navier2D, a MATLAB utility to perform CFD computations [16], which was greatly extended by our research team including data

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collection modules for system identification, as well as extensions to the solvers to perform closed-loop simulations. We first select two small regions at the top and bottom of the cylinder for actuation and assume that we can blow/suck fluid from these locations. The GUI of the Navier2D program and selected actuation and measurement region can be seen in the left hand side of Fig. 1. The actuation holes can be seen as tiny magenta points on the cylinder and the measurement region is the magenta rectangular area towards right of the cylinder.

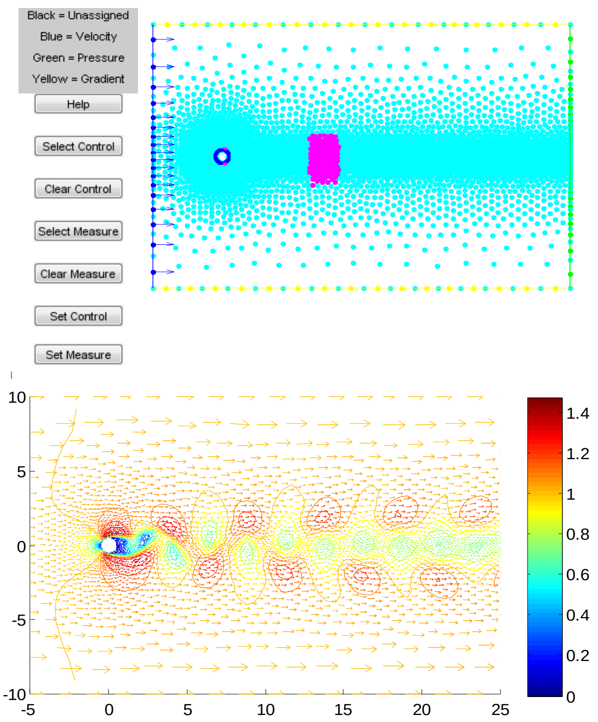

Fig. 1. Selected actuation and measurement points of the Navier2d program (upper) and the velocity field resulting from running the CFD simulation for about 250 seconds (upper).

Science our goal is to suppress the vortex shedding past the cylinder, we select a group of nodes behind it to measure the mean vorticity to be set to a reference value. The vorticity value was calculated using Eq. (1)

$$\eta = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \tag{1}$$

where U and V are the streamwise and transverse components of flow velocity. As the output of the system we take the mean vorticity magnitude over the group of nodes shown in Fig. 1. For the CFD simulations the system was excited for about 250 s at Reynolds number (Re) 150 which is greater than 47 so that repeating pattern of vortex can be seen. The kinematic viscosity value of the fluid is $= 0.0067\ \mathrm{m}^2/\mathrm{s}$. The diameter of the cylinder is $1\ m$, and it is centered at the origin $(0, 0)$ of the $[-5\mathrm{m}, 25m] \times [-10m, 10m]$ sized flow domain. The fluid flows into the domain from the left hand side at a velocity $1\ \mathrm{m/s}$. The surface of the cylinder is assigned no slip boundary conditions (i.e. the U and V components are set to zero) and the top and bottom boundaries assigned as free slip surfaces. (i.e. the derivative of U velocity component over the perpendicular direction to the boundary and the V velocity component are set to zero.)

The next step is to collect the output measurements under a significantly exciting input signal. This input-output dataset will later be used to estimate a dynamical model of the system. To estimate the system dynamics accurately, the input function should contain a variety of frequencies. For this purpose we used a chirp signal of unit magnitude, duration of 50 s, where the frequency varies from 0.1 Hz to 1 Hz for the first 25 s and then goes from 1 Hz back to 0.1 Hz for the next 25 s. The input applied and the output resulting from CFD simulations using Navier2D are shown in Fig. 2.

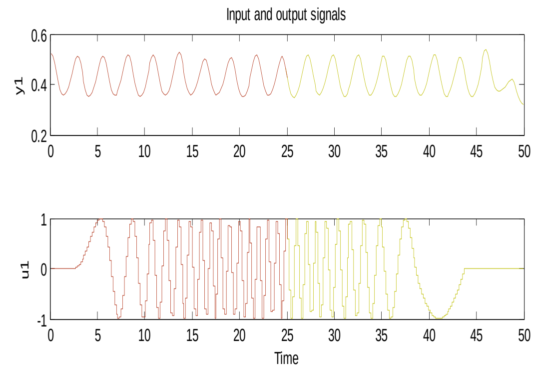

Fig. 2. The Chirp signal input ($u1$) and the resulting measured output ($y1$).

In the Fig. one can see that the input and output signals are partitioned into two parts shown in red and yellow. The red part is *working data*, which is the first 25 s of the data and will be used for building the dynamical model using system identification. The yellow part is the *verification data*, which is the last 25 s of the data and is reserved for validating the results of system identification.

B. System Identification

For the purpose of modeling we seek a Hammerstein-Wiener (HW) type nonlinear model for the input/output data collected previously. In this approach is based on the expectation that if output of the system depends nonlinearly on its inputs it is possible to decompose the input-output relation into two or more interconnected elements [17], [18]. Thus the method represents the nonlinear dynamical model as three serial blocks which are input nonlinearity function block, linear block and output nonlinearity function block. A HW system block diagram can be seen in Fig. 3.

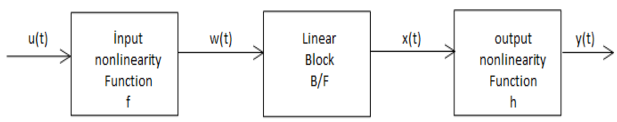

Fig. 3. Block diagram of Hammerstein-Wiener model.

Most system identification tools, including MATLAB System Identification Toolbox that is used in this study, are capable of building HW models with quite general input/output nonlinearity functions. However, we impose additional restrictions on the identification process to benefit the controller design in the succeeding sections. We constrain the nonlinearities to be piecewise polynomials of at most degree three and require that the resulting nonlinear functions be invertible. The input nonlinearity function obtained from system identification is

$$y=\begin{cases}0.13278x-0.4468, x\le -1\\ -0.0131x^3+0.0113x^2+0.4349x+0.0332, -1\le x\le 1\\ 0.24577x+0.37195, x\ge 1\end{cases} \quad (2)$$

and the output nonlinearity function is

$$y=\begin{cases}-0.0031677x+0.46641, x\le -1\\ -7.597610^{-7}x^3+1.069010^{-5}x^2-0.0031x+0.4665\\ 1\le x\le 1\\ -0.0030608x+0.46646, x\ge 1\end{cases} \quad (3)$$

which are plotted is Fig. 5.

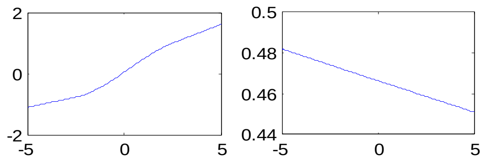

Fig. 4. Graphics of the input nonlinearity function (left) and output nonlinearity function (right).

Also identified is the linear block of the Hammerstein-Wiener model, which is

$$G(s)=\frac{-1.84s^2-14.46s+96.48}{s^3+0.03386s^2+5.59s+0.01893} \quad (4)$$

Finally Fig. 5 shows the full range of the measured data obtained from CFD simulations compared with the output of the HW model identified. It can be observed the HW model captures the general trend of the flow process in the measurement region, which is satisfactory and adequate. (It would be unrealistic to expect a perfect match between a simple finite low-order dynamical ODE model and the complicated infinite order NS PDEs used for CFD.)

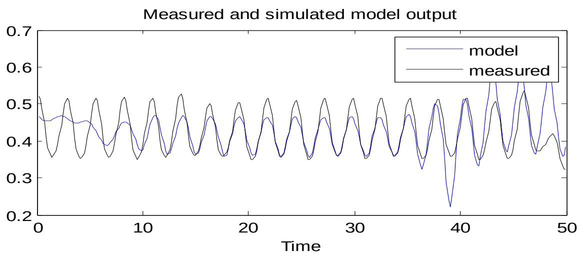

Fig. 5. Comparison of model outputs and measured data from CFD simulations.

C. Controller Design

After obtaining the nonlinear dynamical model of the fluid flow system by using system identification, the next step is the design of the controller. The procedure is to first design a controller for the linear block of the HW model and then to map this controller to the nonlinear model by reversing the input/output nonlinearities. The latter step is possible since these nonlinearities were constrained to be invertible during the identification process. To design the controller, several standard automated tuning methods such as Ziegler–Nichols PID, internal model control (IMC), linear quadratic Gaussian (LQG) and optimization based approaches were tested and the best results were obtained for the LQG design method [19]-[21]. The transfer function of the designed controller is as follows

$$\frac{7.2810^{-7}s^2+5.24710^{-5}s+1.81910^{-6}}{s^2+0.05734s-2.13210^{-16}} \quad (5)$$

Also the closed-loop step response and input amplitude graphics for the linear block is shown in Fig. 6.

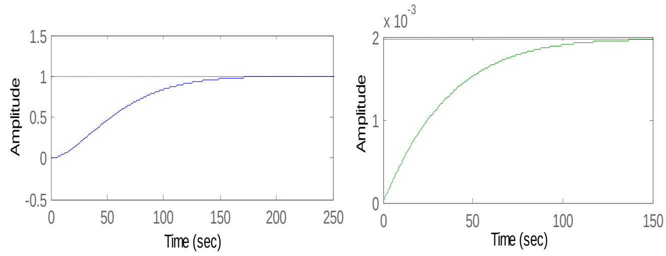

Fig. 6. Closed loop (left) and input amplitude (right) graphics.

D. Integrating the Controller to the Simulation

In this section the integration of the designed controller to CFD simulations in other to achieve closed-loop simulations will be discussed briefly. A block diagram from the controller's perspective is illustrated in Fig. 7.

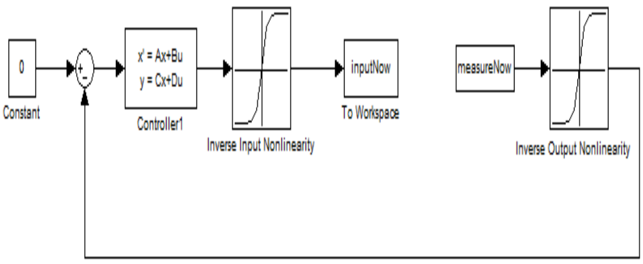

Fig. 7. SIMULINK block diagram for the controller's operation during closed-loop simulations.

After every simulation step, the controller gathers the measured values computed by the CFD solver Navier2D (denoted *measureNow*) and passes this value through the inverse output nonlinearity to map the output to the linear portion of the model. This value is then compared with the desired output value (which is zero, since the ideal case is total suppression of the vortices). This difference is then fed to the controller, which produces an input signal to be applied to the linear block of the model. This value is then mapped to the nonlinear model of the flow process through the inverse input nonlinearity. The result is the value of actuation to be applied to the flow process, which is sent to the CFD solver through a workspace variable (denoted *inputNow*). These steps repeat for every iteration of the CFD simulation.

III. Results

To evaluate the modeling and control approach proposed a closed-loop CFD simulation was carried out for 1000 s. The initial condition for the simulation is the vortex shedding pattern seen in Fig. 2. The vorticity values simulation at times $t = 0.74237$ s, 684.923 s, 871.0875 s and 999.978 s can be seen in Fig. 8.

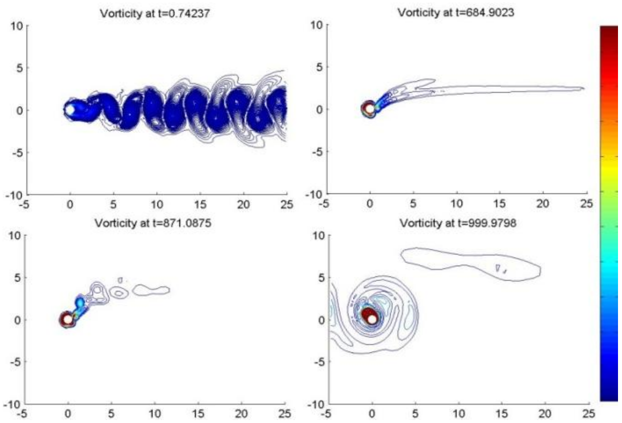

Fig. 8. Vorticity values of the flow field at indicated times.

One can observe that the controller gradually takes effect and finally suppresses the vortices within the desired measurement region behind the cylinder. The controller effort is also clearly visible as the red colored regions near the suction/blowing zones on the cylinder.

IV. Conclusion and Future works

In this study a systematic approach for the nonlinear modeling and control of the flow past a circular cylinder was considered. Input/output data from CFD simulations were collected and were used to identify a nonlinear HW model, where the input/output nonlinearities were made to be invertible. This property was exploited by the control design step, where an LQG controller was designed for the linear block of the HW model, whose input and output were augmented with the reversed nonlinearity functions to map the controller to the nonlinear HW model. CFD simulations for closed-loop system demonstrated that the controller could adequately suppress vortices within a selected measurement zone.

Future works include applying the techniques considered to different flow geometries such as the flow around a square, over an airfoil, through a pipeline and so on.

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